Evidence for the Breakdown of the general Plane-Wave Field Emission Theory in Nanometric Emitters

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It is well-known that neither the elementary triangular barrier Field Electron Emission (FE) equation nor the Murphy-Good FE equation holds at the nanometric scale of emitters. This has been recently supported by the experimental work of Meng and coworkers [1]. Here, we give evidence that even the general FE theory based on plane waves and Cartesian symmetry is not valid. To this end, we have calculated the local electrostatic potential distribution for emitters with apex radii of 2 nm and 4 nm, with a planar anode located at a gap distance of d = 25 nm and 50 nm, respectively. The local electrostatic potential distributions were calculated using finite elements method in a 2D axisymmetric simulation domain in COMSOL by solving the Laplace equation [2]. Subsequently, the local electrostatic potential was used to reproduce the experimental data of Meng *et al.* [1] by employing the general theoretical expression for the local transmitted current density through an arbitrary potential barrier using plane waves as eigenstates. That is, the expression for the local current density J

$$J = \frac{emk_BT}{2\pi^2\hbar^3}\int ln\left[1 + exp\left(\frac{E_F - E_n}{k_BT}\right)\right] D(E_n)\, dE_n \quad (1)$$

where D is the transmission coefficient, E_n are the normal energies of the electrons and the remaining symbols have their conventional meaning. The $D(E_n)$ depend on the tunneling potential distribution $U(x)$ which is constructed by adding a spherical image potential and the work function to the calculated complete electrostatic potentials i.e. no linearity in the potential was assumed (like in the elementary or in the Murphy-Good FE equations) or even corrections up to second order like in the Kyritsakis-Xanthakis equation [3] Then, in normal practice, the current is I=J*A where A is the area which is treated as a parameter. The calculated slopes of the ln(I) vs $1/V$ curves – (where V is voltage) - deviated significantly from the experimental ones signifying that the true nature of the eigenstates in these nanometric emitters due to confinement and localization must be taken into account. We note that A cannot affect the slopes of these curves. By describing the emitters as paraboloids [4] i.e. by using wavefunctions with wavefronts that fit the emitter surface we have managed to reproduce the experimental data of the R=2 nm emitter. Calculations to reproduce the 4 nm emitter I-V characteristics are under way.

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